J. Fluid Mech. (2007), *vol.* 582, *pp.* 473–475. © 2007 Cambridge University Press
Printed in the United Kingdom

BOOK REVIEW

Mathematics of Large Eddy Simulation of Turbulent Flows. By L. C. Berselli, T. Iliescu & W. J. Layton. Springer, 2006. 348 pp. ISBN 987 3 540 26316 6. 74.85 €

J. Fluid Mech. (2007), *vol.* 582, doi:10.1017/S0022112007006477

It is a difficult undertaking to write a book on Large-Eddy Simulation (LES) nowadays. The field is still developing so fast that it is hard to identify milestones which could serve to limit the content to be considered in a book. After it appeared that LES development had settled on a *modus operandi* similar to Reynolds-averaged Navier–Stokes (RANS) modelling with the 'established' range of 'robust' models, for example the dynamic Smagorinsky model or the structure-function model, attempts to extract more information on the resolved scales and a closer analysis of the interference between numerical discretization and subgrid-scale (SGS) models along with SGS modelling errors have stimulated a new development in the last decade.

There are a number of recommendable books on LES. In particular that by Sagaut should be mentioned (*Large Eddy Simulation for Incompressible Flows*, Springer, 2001) which in a short time has become a standard reference for the LES modeller. Whereas engineers have been the motor driving LES modelling, rather few mathematicians have taken up the burden to follow in the tracks and clean out the rubble. The authors of the book under review are a particular successful group of mathematicians addressing this task, and they never lose sight of the application and the empirical LES modelling.

In the following a very brief summary of the book chapters from the perspective of the engineer or physicist dealing with LES modelling is given.

Chapter 1 summarizes common facts about flow physics, the closure problem in turbulence modelling, some basic problems in LES and established SGS models. The section on validation and testing is perhaps a little 'handwaving' in nature and of only moderate help for a practitioner.

Chapter 2 deals with the mathematical properties of the incompressible, single-phase Navier–Stokes equations. The derivation of the NSE as in section 2.2 is, however, merely a statement of the results of the physical derivation of the NSE based on the Reynolds-transport theorem. A definition of weak solutions of the Navier–Stokes equations along with a summary of mathematical tools is provided. The chapter closes with a brief discussion of the stochastic Navier–Stokes equations.

Chapter 3 introduces the closure problem in LES and provides a review of the eddy-viscosity or Smagorinsky subgrid-closure along with its well-known and not so well-known variants, the latter based on the theoretical work of the books' authors. For an extended formulation of the Smagorinsky model an existence and uniqueness proof for the filtered solution is given.

Chapter 4 includes a brief summary of other models based on the eddy-viscosity approach. Some emphasis is given to the Prandtl-type eddy-viscosity estimate based on a Pade-approximation of the filter inverse. Also, two-equation models and corrections to eddy-viscosity models are briefly addressed.

A topical issue is addressed briefly in chapter 5, dealing with modelling uncertainties. Approaches on how to analyse SGS-model uncertainties are outlined using the example of eddy-viscosity models.

Eddy-viscosity models having been reviewed, in chapter 6 the reader is guided towards models which attempt to reproduce some of the subfilter-scale structure. Requirements of the model are proposed, such as realizability and reversibility. Here it should be mentioned that dynamic eddy-viscosity models (without *ad hoc* modifications) are reversible, as shown by Carati *et al.* (*J. Fluid Mech.*, vol. 441, 2001, p. 119). Also, other mathematical (consistency, symmetries) and physical constraints (validation requirements for the usual canonical cases) are formulated.

Chapter 7 is devoted to closures based on filter and inverse-filter approximations in Fourier dual space. The first model investigated on that basis is the gradient or tensor-diffusivity model. For this model, supplemented by an eddy-viscosity term, an existence and uniqueness result is shown. Numerical evidence supports the conclusion that the gradient model can be unstable unless a sufficiently dissipative correction is added. The next model considered in this family is the rational model, developed by the research group of the authors. The essence of the model is a rational approximation in Fourier dual space for the filter transfer function. An alternative version obtained by including an eddy-viscosity in the rational model is also discussed. Under restrictive assumptions the existence and uniqueness for the solution of the rational model is proven. Criteria for the breakdown of strong solutions are given. An added eddy-viscosity can result in energy stability where the necessary dissipation is weaker than for the gradient model. At the end of the chapter a higher-order extension of the rational model is introduced and discussed. *A priori* tests on wavelike solutions show a significant improvement compared with the gradient model.

The last chapter on SGS models, chapter 8, deals with scale-similarity models. The starting point is the model of Bardina which has initiated most further developments on these so-called structural modelling ideas. The model has never been used successfully in computations without additional regularization. To explain this, the authors give an argument which essentially says that a sufficiently dissipative term needs to be added to compensate for an instability arising from a third power of the norm of the gradient of the solution. Further members of the scale-similarity family are outlined. Although formally an eddy-viscosity model, the dynamic model involves a dynamic constant which allows the model to be interpreted as a scale-similarity approach. Some space is devoted to skew-symmetric model formulations, one of them exhibiting favourable theoretical properties. Nevertheless, these models have not found widespread use so far. The list of higher-order consistent scale-similarity models is introduced by the second-order (in terms of filter scale) scale-similarity model. The further analysis is based on a differential filter which has convenient properties. It is shown that this second-order model is energy stable, and the question is how to preserve this stability while increasing the consistency order. For this purpose the approximate deconvolution model (ADM) is used and it is shown that under the condition that the deconvolution operator be positive higher-order schemes obtained by ADM are energy stable. The chapter closes with an outlook on refined deconvolution procedures that may lead to further improved models.

Chapter 9 is devoted to the commutation error for filter and derivative operators in the case of inhomogeneous filter kernels. A key result is the derivation of a boundary commutation error which poses a second modelling issue. Common practice with LES is to lump commutations errors with the modelling errors rather than consider them specifically. Commutation errors can be reduced with deconvolution methods. In that case the filtered solution assumes properties of the filter, which has to taken into account when analysing results. The same holds for finite-volume methods, and – as the authors point out – for finite-element methods. Although theoretically important, in practice most of the developments in treating commutation errors have not found their way into applications.

Chapter 10 deals with the important issue of wall modelling. While mentioning some previous and current approaches not much is said about their analytical or numerical properties. More space is devoted to the discussion of a Navier boundary condition (slip related to friction) for which modelling parameters can be derived for analytical mean-flow profiles. Some statements, such as that boundary conditions should be provided at the physical boundary and not at a virtual boundary within the flow, act as guidelines for the practitioner.

Chapter 11 attempts to provide a link between the purely analytical results of the previous chapters and the necessary final step to numerical discretization. Certainly, given the mathematical apparatus, the most straightforward approach is to focus on function-space discretizations. Unfortunately, this is rarely used in CFD practice. The link between numerics and the previously discussed models is yet incomplete. Mainly, the variational multi-scale model is mentioned, which has not yet found widespread use in the community either.

In chapter 12 some selected models, applied to turbulent channel flow at moderate friction Reynolds numbers, are compared. Considered are the rational model, the gradient model and the Smagorinsky model with wall damping; discretization is with a spectral-element method. The results fall into the standard range of what is currently observed. They appear to be worse than can be obtained with less sophisticated finite differencing and the dynamic model, but a firm conclusion is difficult since only limited details on the numerical setup are provided.

In conclusion, the authors have written a book which provides a mathematical background on current large-eddy-simulation modelling approaches. While biased towards the authors' own developments, other approaches are also discussed extensively. It is not necessarily a book for the practitioner, but by skipping over the mostly fully provided proofs a researcher on LES model development will find a wealth of useful information. For a second edition one would like to see more space devoted to the relation between SGS models and numerical discretizations, if possible including finite-volume and perhaps even finite-difference schemes. The results section certainly appears currently as a 'work in progress' and could be more comprehensive in the future.

Nikolaus Adams